Multigrid Exercise

- Use a multigrid algorithm to solve the Poisson equation

$$\begin{aligned} -\Delta u &= f \qquad \text{on } \Omega = (0,1)^2 \\ u &= 0 \qquad \text{at } \partial\Omega \end{aligned}$$

1. Compute $f(x, y)$, such that $u(x, y) = g(x, y) \sin \pi x \sin \pi y$ is a solution for some choice of the the smooth function $g(x, y)$.

2. Use a finite difference discretisation to discretize the Poisson equation and write this as a linear system.

3. Write a multigrid program to solve the linear system

   (a) Define as sequence of meshes

   (b) Use the damped Jacobi method as smoother, full weighting for the restriction and linear interpolation for the prolongation.

4. First test your program on a coarse mesh with $g(x, y) = 1$

5. Next, test your program using two meshes and finally for four meshes. Evaluate the effect of ω on multigrid. Is $\omega = 4/5$ optimal?

6. Plot multigrid convergence as function of the number of iterations.

7. Test multigrid algorithm also for the Gauss-Seidel smoother with lexicographic ordering and repeat the tests you did for damped Jacobi.