电磁学 A 作业一

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Problem 0

自学绪论部分的课件.

Problem 1

证明下列等式:

1. $(\vec{A}\times\vec{B})\times\vec{C}=(\vec{A}\cdot\vec{C})\vec{B}-(\vec{B}\cdot\vec{C})\vec{A}$
2. $\vec{A}=(\hat{n}\cdot\vec{A})\hat{n}+(\hat{n}\times\vec{A})\times\hat{n}$,

其中 $\hat{n}$ 为任意方向单位矢量.

Problem 2

证明下列公式:

1. $(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D})=(\vec{A}\cdot\vec{C})(\vec{B}\cdot\vec{D})-(\vec{A}\cdot\vec{D})(\vec{B}\cdot\vec{C})$
2. $\vec{A}\times[\vec{B}\times(\vec{C}\times\vec{D})]=\vec{B}[\vec{A}\cdot(\vec{C}\times\vec{D})]-(\vec{A}\cdot\vec{B})(\vec{C}\times\vec{D})$
3. $[\vec{A}\times(\vec{B}\times\vec{C})]+[\vec{B}\times(\vec{C}\times\vec{A})]+[\vec{C}\times(\vec{A}\times\vec{B})]=0$

Problem 3

1. 计算标量场
$$\varphi=-\frac{1}{r}$$
的对应的梯度 $\vec{F}$.

2. 证明矢量场 $\vec{F}$ 的散度和旋度分别满足:

$$\nabla \cdot \vec{F} = 0 \quad (r \neq 0)$$
$$\nabla \times \vec{F} = 0$$

Problem 4

计算下列矢量场的散度和旋度:

1. $$\vec{E}_1 = \cos(z)\hat{x} + \sin(z)\hat{y} + z^2\hat{z}$$

2. $$\vec{E}_2 = A\frac{e^{-br}}{r}\hat{r}$$

3. $$\vec{E}_3 = A\ln R\hat{z}, \quad R = \sqrt{x^2 + y^2}$$

4. $$\vec{E}_4 = \nabla \times \vec{E}_3$$

Problem 5

证明以下 ∇ 算符的运算公式:

$$\nabla(\varphi\phi) = \phi\nabla\varphi + \varphi\nabla\phi, \tag{1}$$
$$\nabla \cdot (\varphi\vec{A}) = \varphi\nabla\vec{A} + \vec{A} \cdot \nabla\varphi, \tag{2}$$
$$\nabla \times (\varphi\vec{A}) = \varphi\nabla \times \vec{A} + \nabla\varphi \times \vec{A}, \tag{3}$$
$$\nabla(\vec{A} \cdot \vec{B}) = \vec{A} \times (\nabla \times \vec{B}) + (\vec{A} \cdot \nabla)\vec{B} + \vec{B} \times (\nabla \times \vec{A}) + (\vec{B} \cdot \nabla)\vec{A}, \tag{4}$$
$$\nabla \cdot (\vec{A} \times \vec{B}) = (\nabla \times \vec{A}) \cdot \vec{B} - \vec{A} \cdot (\nabla \times \vec{B}), \tag{5}$$
$$\nabla \times (\vec{A} \times \vec{B}) = (\vec{B} \cdot \nabla)\vec{A} + (\nabla \cdot \vec{B})\vec{A} - (\vec{A} \cdot \nabla)\vec{B} - (\nabla \cdot \vec{A})\vec{B}, \tag{6}$$
$$\nabla \cdot (\nabla\varphi) = \nabla^2\varphi \tag{7}$$
$$\nabla \times (\nabla\varphi) = 0, \tag{8}$$
$$\nabla \cdot (\nabla \times \vec{A}) = 0, \tag{9}$$
$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2\vec{A}, \tag{10}$$

(4)(6) 可选做。

Problem 6(选做)

推导矢量场的散度、旋度、标量场的梯度在柱坐标系下的可以写成以下形式：

$$
\begin{aligned}
\nabla \cdot \vec{A} &= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho A_\rho\right) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \\
\nabla \times \vec{A} &= \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\rho} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\phi} + \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}\left(\rho A_\phi\right) - \frac{\partial A_\rho}{\partial \phi}\right]\hat{z} \\
\nabla \Phi &= \frac{\partial \Phi}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial \Phi}{\partial \phi}\hat{\phi} + \frac{\partial \Phi}{\partial z}\hat{z}
\end{aligned}
\tag{11}
$$

其中 $\vec{A} = A_\rho\hat{\rho} + A_\phi\hat{\phi} + A_Z\hat{z}$.

Problem 7(选做)

推导矢量场的散度、旋度、标量场的梯度在球坐标系下的可以写成以下形式：

$$
\begin{aligned}
\nabla \cdot \vec{A} &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(A_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi} \\
\nabla \times \vec{A} =& \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}\left(A_\phi \sin\theta\right) - \frac{\partial A_\theta}{\partial \phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}\left(rA_\phi\right)\right]\hat{\theta} \\
&+ \frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\theta\right) - \frac{\partial A_r}{\partial \theta}\right]\hat{\phi} \\
\nabla \Phi &= \frac{\partial \Phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial \Phi}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial \Phi}{\partial \phi}\hat{\phi}
\end{aligned}
$$

其中 $\vec{A} = A_r\hat{r} + +A_\theta\hat{\theta} + A_\phi\hat{\phi}$.