

Arithmetic Groups and Diophantine Geometry

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ALGANT: Leiden (2013 - 2014)

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f is holom. at the cusps. $f(\frac{az+b}{cz+d}) = (cz+d)^k f(z)$
 let $n \geq 5, k \in \mathbb{Z}$. Then $M_k(\Gamma_1(n)) = H^0(X_1(n), \omega^{\otimes k})$
 heaf of sect of $\mathbb{C} \times \mathbb{H}$ $(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}) : (x, z) \mapsto ((cz+d)^k x, \frac{az+b}{cz+d})$
 cusp forms. $S_k(\Gamma_1(n)) := \{f \in M_k(\Gamma_1(n)) : f \text{ is zero at every cusp}\}$
 $(a_0(f) = 0)$

For $E/\mathbb{C}$ ell. curve, $\omega_E = T_E(o)^\vee$ 1-dim $\mathbb{C}$ -vec sp.
 $\omega_E = \{f : (E, P) \mapsto f(E, P) \in \omega_E\}$
 s.d.: f vanishes holom. with (E, P)
 $\exists$ isom. $\varphi : (E, P) \rightarrow (E', P')$
 $\exists$ f vanishes on the cusps.
 $T_\varphi : T_{E(o)} \rightarrow T_{E'(o)}$
 $\omega_E \xrightarrow{\varphi^*} \omega_{E'}$
 $f(E, P) \xrightarrow{\varphi^*} f(E', P')$
 $f(\mathbb{C}/(\mathbb{Z} + \mathbb{Z}z), \frac{1}{n}) = F_f(z) (dz)^{\otimes k}$
 $= F_f(z) (dx)^{\otimes k}$ holom

Hecke operators $S_k(\Gamma_1(n))$
 Diamond op: $Vac(\mathbb{Z}/n\mathbb{Z})^*$
 $(\alpha)(E) = \pm k \alpha \cdot n$
 $V_m \geq 1$
 $V_r \geq 1$
 $\forall f \in S_k(\Gamma_1(n))$
 $a_m(T_r f) = \sum_{\substack{d|n, m \\ (d, n)=1}} d^{k-1} a_{m/d}(f)$

Consequences:
 1. The $T(n, k)$ all commute
 2. the (α) are in the alg. $T(n, k) \subset \text{End}_{\mathbb{C}} S_k(\Gamma_1(n))$
 generated by
 3. $\sum_{n \geq 1} T_n$
 4. $a_1(T_r)$









